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Discussion of "Of quantiles and expectiles: consistent scoring functions, Choquet representations and forecast rankings" by W. Ehm, T. Gneiting, A. Jordan and F. Krüger

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We congratulate the authors for this thought-provoking lesson for forecasters. In the space available we focus on discussing the possibility of using summary measures based on Murphy diagrams for suggesting 'optimal' ways of combining forecasts. In principle one would expect that in many settings of applied interest the performance of competing forecasters would be more like the inflation example in Section 4.1, where the SPF dominates for some values of $\Theta = \{x_{11}, x_{12}, y_1, \ldots, x_{n1}, x_{n2}, y_n\}$ but not on others. Typically in cases where there is no clear cut forecast dominance, one could wonder how does the Murphy diagram of forecast combinations compares. For example, how does the Murphy diagram of the average of both forecasts, $x_{i3} = x_{i1}/2 + x_{i2}/2$, compares with the SPF (x_{i1}) and Michigan (x_{i2}) forecasts? As can be seen in Fig. 1 (a), the average of forecasts performs better on some values on some regions of Θ but not on others. One could ask: "Is there any other convex combination performing 'better'? How to define 'better' in terms of the Murphy diagram?" To approach these questions consider the forecast combination $x_{i3}(w) = wx_{i1} + (1-w)x_{i2}$, and—extending ideas from Section 3.3—define the *area under the Murphy diagram* and the *maximum of the Murphy diagram* respectively as

$$A(w) = \int_{\theta^-}^{\theta^+} s_3(\theta, w)\,\mathrm{d}\theta, \quad B(w) = \max_{\theta \in [\theta^-, \theta^+]} s_3(\theta, w),$$

where θ^- and θ^+ respectively denote the min and max of $\{x_{11}, x_{12}, x_{13}, y_1, \ldots, x_{n1}, x_{n2}, x_{n3}, y_n\}$, and $s_3(\theta, w) = n^{-1}\sum_{i=1}^n S_\theta(x_{i3}(w), y_i)$ with $w \in [0,1]$. Smaller values of these summaries of the Murphy diagram are compatible with a good forecast accuracy. Indeed, if there was a value of $w = w'$ for which the combination of forecasts coincided with the data, then $A(w') = 0$ and $B(w') = 0$. Thus, a natural way of defining the 'best' convex linear combination of forecasts using Murphy diagrams is as $x_{i3}(w^*_\cdot)$, where $w^*_A = \arg\min_{w\in[0,1]} A(w)$, or through the minimax criteria $w^*_B = \arg\min_{w\in[0,1]} B(w)$. We call to $x_{i3}(w^*_\cdot)$ as a *Murphy optimal combination forecast*. For example, for the inflation forecasts $w^*_A = 0.65$ and $w^*_B = 0.94$; also, $A(w^*_A) = 0.38$, whereas $A(1) = 0.41$ (SPF), $A(0) = 0.49$ (Michigan), and $A(1/2) = 0.39$ (mean of forecasts); in addition, $B(w^*_B) = 0.162$, $B(1) = 0.163$, $B(0) = 0.195$, and $B(1/2) = 0.176$. See Fig. 1 (b) and (d) for the plots of $A(w)$ and $B(w)$ over the $[0,1]$ interval.

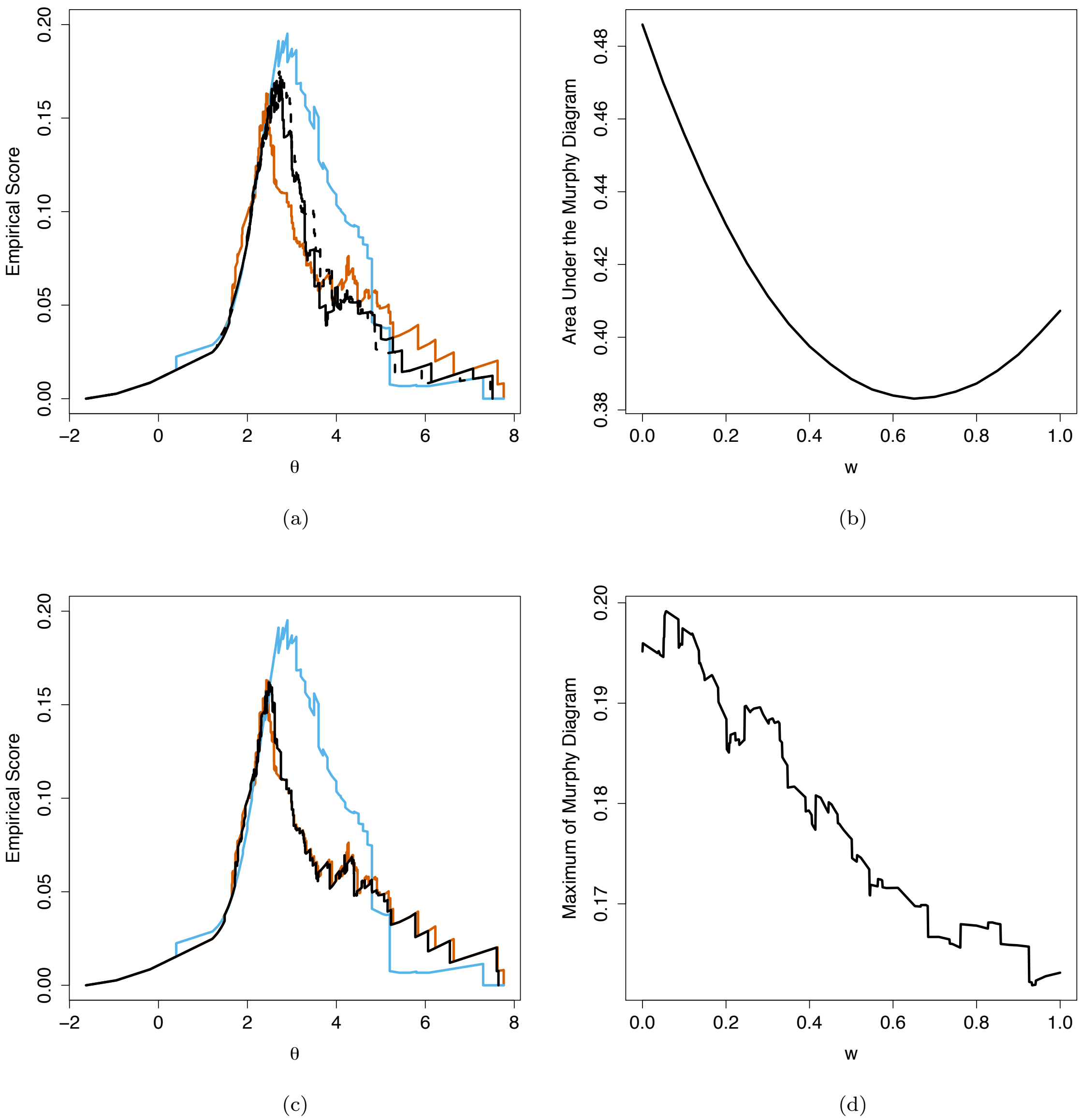

Figure 1: (a) Murphy diagrams for inflation example (orange: SPF; blue: Michigan; dashed black: average combination forecast; solid black: Murphy optimal combination forecast, $w_A^* = 0.65$). (b) Area under the Murphy diagram. (c) Murphy diagrams for inflation example (orange: SPF; blue: Michigan; solid black: Murphy optimal combination forecast, $w_B^* = 0.94$). (d) Maximum of the Murphy diagram.